# UNIVERSITY COLLEGE LONDON

University of London

## EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

*B.Sc.* *B.Sc.(Econ)M.Sci.*

**Mathematics M14B: Mathematical Methods 2**

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|---|---|---|
| COURSE CODE | : | **MATHM14B** |
| UNIT VALUE | : | **0.50** |
| DATE | : | **05-MAY-05** |
| TIME | : | **14.30** |
| TIME ALLOWED | : | **2 Hours** |

05-C0960-3-210



**TURN OVER**

*All questions may be attempted but only marks obtained on the best* ***four*** *solutions will count.*
*The use of an electronic calculator is* ***not*** *permitted in this examination.*

1. (a) State, without proof, the general formula for a Fourier series on $(-L, L)$, with $L > 0$, for a function $f(x)$, giving the expressions for the coefficients.
   (b) On $(-\pi, \pi)$, find the Fourier series of $f(x) = |x|$.
   (c) State Parseval's identity.
   (d) Apply Parseval's identity to the function of part 1b to obtain an infinite series for a power of $\pi$.

2. (a) Using subscript notation, what is the expression for

$$\varepsilon_{ijk}\varepsilon_{klm}$$

   in terms of $\delta_{il}$, $\delta_{jm}$, etc.?
   (b) Using subscript notation, prove

$$\text{curl}\,(\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - \mathbf{B}(\,\text{div}\,\mathbf{A}) - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\,\text{div}\,\mathbf{B}).$$

   (c) Verify the result of part 2b for

$$\mathbf{A} = (1, 0, 0), \qquad \mathbf{B} = (x, y, z).$$

3. (a) Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = (2x - y, 2y - x, 0)$ and $C$ is the perimeter of the ellipse $x^2/9 + y^2/4 = 1$, $z = 0$, described in the anticlockwise sense.
   (b) Find $\int \mathbf{G} \cdot d\mathbf{r}$ for $\mathbf{G} = (y, z, yx)$ from $(0, 0, 0)$ to $(1, 1, 1)$ along
      (i) $\mathbf{r} = (t^2, t^3, t)$ with $0 \leq t \leq 1$,
      (ii) $\mathbf{r} = (t, t^2, t)$ with $0 \leq t \leq 1$.

   Is $\mathbf{G}$ conservative? Give your reasons for your conclusion.

4. (a) Define the Jacobian

$$\frac{\partial(x,y,z)}{\partial(u,v,w)},$$

where $x(u,v,w)$, $y(u,v,w)$ and $z(u,v,w)$ are smooth functions.

(b) Illustrate the cylindrical polar coordinate system by means of a sketch, and give the expressions for the Cartesian coordinates $(x,y,z)$ in terms of the cylindrical polar coordinates.

(c) Determine the Jacobian, from your definition, for the change of coordinates from Cartesian coordinates to cylindrical polar coordinates.

(d) Find, by a using a suitable change of coordinates or otherwise,

$$\int_V (1+z^3)\exp(x^2+y^2)\,dV,$$

where $V$ is the region $x^2+y^2 \leq 1$ and $-1 \leq z \leq 1$.

5. (a) State the divergence theorem carefully.

(b) Given

$$\phi = \frac{1}{|\mathbf{r}|}, \qquad \mathbf{E} = -\operatorname{grad}\phi,$$

show that if $|\mathbf{r}| \neq 0$ then $\operatorname{div}\mathbf{E} = 0$.

(c) Let $V_a$ be the ball of radius $a > 0$, and find the value of

$$I \equiv \int_{V_a} \operatorname{div}\mathbf{E}\,dV,$$

with $\mathbf{E}$ as defined in 5b, by first transforming this integral into a surface integral. Explain why the result of part 5b implies that the integral I does not depend on $a$.

6. (a) State Green's theorem carefully.

(b) Verify Green's theorem for the functions

$$P = xy, \qquad Q = (1+x+y)^2,$$

and the region defined by

$$x \geq 0, \quad y \geq 0, \quad x+y \leq 1.$$

Sketch the region of integration.



END OF PAPER